Specialization under Uncertainty

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Abstract

We analyze a general equilibrium model with two sectors, sector-specific skills, and stochastic sector-specific productivity shocks. The main focus of this paper is the choice of specialization by the workers. That is: How much sector-specific human capital should a worker acquire? We identify three reasons for less than perfect specialization: 1) risk-aversion, 2) decreasing returns in human capital accumulation, and 3) substitutability/complementarity between outputs. For a simple distribution of shocks, where the realization of the shocks can take one of two values and the shocks are perfectly negatively correlated, we show that there are always some workers who fully specialize in a competitive equilibrium. Furthermore, if the productivity shocks have large enough variance, there will be some workers who acquire both skills.

We prove that the competitive equilibrium is generally inefficient, and generates too little specialization compared to the social optimum where the social planner can use transfers among the workers. We also argue that if the planner is not allowed to use these transfers, there will be less specialization in this constrained optimal outcome than in the competitive equilibrium. In order to see how the correlation between the productivity shocks affects the specialization by the workers, we compute the equilibrium skill distribution numerically.

1 Introduction

The basic trade-off that arises when an individual determines the structure of human capital to acquire is the trade-off between productivity and mobility. Highly specialized workers are more productive in the area of expertise, but are also bound to this area through the best and the worst. In absence of appropriate insurance, wage fluctuation may induce risk-averse agents to

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forego specialization in favor of a broader scope of skills that would allow them to switch to a better paying job in case of a bad wage shock.

We consider a general equilibrium model where there are two sectors (industries) producing two different goods using sector-specific skills. Agents value goods produced in the two sectors according to a constant elasticity of substitution (CES) utility function. Uncertainty comes into the economy through exogenous productivity shocks that result in endogenous variation in wages and output prices in the two sectors. Workers are ex-ante identical and risk-averse. They have to choose their skill combination before the shocks are realized. They decide which sector to work in after they receive their education and observe productivity shocks in the two sectors.

We characterize a competitive decentralized equilibrium in this model. For a simple case of shocks distribution we prove that the competitive equilibrium allocation always involve some workers who fully specialize (acquire only one sector skill), and for sufficient shock variation there are some workers who acquire both skills (unless the elasticity of substitution between two goods in the agents' utility functions is exactly one). We show that when the elasticity of substitution is above (below) one, individuals who acquire both skills work in the good-shock (bad-shock) sector. That is, in the economy with ex-ante identical workers, risk-aversion generates a non-degenerate distribution of workers over skill types.

Further, we consider the first best (where transfers among workers can be used) and the constrained optimum (where no transfers are feasible). We conclude that the competitive equilibrium is generally inefficient and generates too little specialization compared to the first-best allocation. In addition, the constrained optimum results in even less specialization than the competitive equilibrium. The intuition is that in absence of transfers, the planner can improve upon aggregate welfare (though this is not necessarily a Pareto improvement) because he internalizes the fact that more specialization results in a higher wage variation for all specialized workers.

In addition to the analytical results for the simple case, we numerically solve for equilibrium skill distribution for more general distributions of productivity shocks. We find that different shocks distributions can generate equilibrium skill distributions of very different shapes. In particular, uniform perfectly negatively correlated shocks result in a skill distribution with a "hump" around the middle of the unit interval (i.e., around full generalization point), while uniform i.i.d. shocks generate a skill distribution with two "humps", so that no workers perfectly generalize.

Finally, we analyze two modifications of the model, where we introduce capital and allow the capital supply decision to be endogenous. First, we suppose that prior to shocks realization workers can choose in which sector to supply their capital. We show that by investing in the sector opposite from the one they acquire skill in, workers can eliminate the insurance problem partially, but not completely as long as labor income is a substantial part of their earnings (in particular, when labor's share exceeds one half), and thus there will be still less than perfect specialization in such an economy. Second, we consider a model where capital supply decisions can be made after the productivity shocks are realized. We show that in this case an even smaller variation in shocks is needed for less than perfect specialization than in the original model (without capital mobility). Intuitively, capital flows into the more productive sector which decreases the wage in the less productive sector even further, making it less attractive to take risk by becoming a specialist. If we view the skill-acquirement decision as a life-time choice of profession, then it is quite sensible to assume that the capital investment decision is relatively short-term, suggesting that the second extension is more relevant.

This paper fits into the strand of "increasing returns" literature. This literature probably started with Adam Smith (1776) who suggested that specialization results in higher productivity due to the following three factors: (1) frequent repetition of a single task results in improving dexterity, (2) moving from one task to another takes time, and (3) it is easier to invent a time-saving machine for a specialized task. In a more recent work, Rosen (1983) emphasized a fixed-cost nature of human capital investment, which implies that more intensive use of a skill results in increasing returns to the investment.

A number of factors that may limit specialization has been studied. Adam Smith (1776), and later Baumgardner (1988) and Kim (1989), argue that the extent of the market imposes a natural limit. The greater the market, the higher the likelihood to find a contract matching specialized skills. Becker and Murphy (1992) emphasize importance of interaction among specialized workers. Combining these workers into one firm can be associated with principal-agent conflicts, hold-up problems and other similar costs that increase with the extent of specialization. Finally, Murphy (1986) studies a model with exogenous stochastic shocks to demand for goods produced in two sectors, and argues that uncertainty may result in inefficiently low specialization when workers are risk-averse. The model presented in this paper is in many dimensions similar to his, but done in a general equilibrium framework, that allows us to study some new insights.

The paper is organized as follows. In the next section we set up the model and compare the degree of specialization under the competitive decentralized equilibrium, the first best, and the constrained optimum allocations for a simple version of the shocks distribution. In Section 3 we numerically compute the decentralized equilibrium skills distribution for more general distributions of productivity shocks. Section 4 contains brief analysis of the modifications of the model with the capital supply decision made endogenous. Section 5 concludes.

2 The Model

We study a static model of an economy with two sectors, where labor is the only productive input. The production function in each sector is given by $Y_j = z_j L_j^{\alpha}$, where L_j is the total amount of effective labor employed, and z_j is a productivity shock in sector j. Productivity shocks are distributed according to a joint density function $f(z_1, z_2)$. We assume there is a single firm in each sector that maximizes profits taking prices and wages as given, and that the profits are then distributed equally among the households.[1] This last assumption (of profits being distributed *equally*) is crucial – it implies that workers cannot choose their portfolios to provide themselves with an additional form of insurance against risk. In order to study specialization as a sole form of insurance, we shut down this market. In Section 4 we return to this question and explore the case when workers purchase shares (or, alternatively, supply capital) strategically.

There is a continuum of ex-ante identical workers of the total mass one in the economy. Each worker cares about expected von Neumann-Morgenstern utility of consumption of two goods produced in two sectors, with utility in each state being equal to $u\left(v(c_1, c_2)\right) = \left[v(c_1, c_2)^{1-\theta}\right] / (1-\theta)$, where $v(c_1, c_2) = \left(c_1^{(\psi-1)/\psi} + c_2^{(\psi-1)/\psi}\right)^{\psi/(\psi-1)}$. That is, in each state preferences over the two goods are CES with the elasticity of substitution equal ψ, and the utility across the states is also CES, with the elasticity of substitution equal $\frac{1}{\theta}$ (θ is the coefficient of relative risk aversion). Workers are endowed with one unit of pre-market time which they convert into skills, and one unit of productive time which they supply at the labor market.[2] There are two skills, one specific to each sector, and a skill productive in one sector is completely unproductive in the other. The skills affect a worker's productivity multiplicatively. That is, a worker who has σ_j of sector j skill and supplies l units of time in this sector, supplies $\sigma_j l$ units of effective labor. We assume the skill conversion technology (or human capital production function) to be $\sigma_j = t_j^{\gamma}$, where t_j is the pre-market time input directed towards sector j skill, and $\gamma \leq 1$.

[1] We need profits to be returned to households in order to study general equilibrium effects in this economy. Equivalently, we can assume that each household owns two units of a fixed production factor, e.g., land, so that one unit is allocated in each sector. Then the profits that households receive can be viewed as income from renting land. The firms maximize expected profits taking wages and the rental price of land as given, and have technology which is Cobb-Douglas in the two factors. Then constant returns to scale imply that the number of firms does not matter, so that we can consider a single firm in each sector.

[2] We separate the two time endowments for analytical simplicity. Endogenizing the schooling vs. working margin is fairly trivial in this model, and does not bring any new interesting insights into the analysis.

2.1 Decentralized Equilibrium

We first study a competitive decentralized equilibrium in which each worker maximizes his expected utility subject to his budget constraint. Each worker's income comes from his wage earnings plus the firms' profits. The workers' decision process can be considered in three stages. First, they decide on the composition of their skills by allocating their pre-market time between acquiring the two skills. Second, the uncertainty about each sector's productivity is realized, and the workers supply their labor in the sector where it is more productive, given the shocks and their first-stage decision. Third, given the earned income, they decide how to split it between consumption of the two goods. Each worker's problem can be written in the following way:

$$\begin{aligned} \max_{t} \quad & \int \frac{v\left(c_1(z_1,z_2), c_2(z_1,z_2)\right)^{1-\theta}}{1-\theta} f(z_1,z_2) d(z_1,z_2) \\ \text{s.t.} \quad & y(z_1,z_2) = \max\left\{t^{\gamma} w_1(z_1,z_2), (1-t)^{\gamma} w_2(z_1,z_2)\right\} + \pi(z_1,z_2), \\ & p_1 c_1(z_1,z_2) + p_2 c_2(z_1,z_2) = y(z_1,z_2). \end{aligned}$$

The wages and profits, taken as given by the workers, are $w_j = \alpha p_j z_j L_j^{\alpha-1}$, $\pi_j = p_j Y_j - w_j L_j = (1-\alpha) p_j z_j L_j^{\alpha}$, $j = 1,2$, $\pi = \pi_1 + \pi_2$.

The skill combination that a worker acquires on the first stage can be viewed as a location on a unit interval, where being located at point x means that you spent a fraction $(1-x)$ of your time on sector 1 skill, and the remaining fraction x on sector 2 skill. In particular, being located at zero (one) means that a worker fully specializes in sector 1 (sector 2) skill. Throughout the whole paper we assume that ex-ante the two sectors look exactly identical, so that the skill distribution will be symmetrical as well. The generalization to a not symmetrical case is straightforward.

To better understand the mechanics of the model, we consider a special case with the following distribution of productivity shocks:[3]

$$f(z_1,z_2) = \begin{cases} \frac{1}{2}, \text{ if } (z_1,z_2) = (A,1), \\ \frac{1}{2}, \text{ if } (z_1,z_2) = (1,A), \\ 0, \text{ otherwise}, \end{cases} \tag{1}$$

where $A \leq 1$ is a constant. That is, there are two equally possible states of the world, one corresponding to a productivity boom in sector 1, and the other is completely symmetric with sector 2 booming. The degenerate case with $A = 1$ corresponds to full certainty about productivities.

For convenience let us normalize the price of output in the good-shock sector to 1, and denote the price of output in the bad-shock sector by p. It immediately follows that the solution of the

[3] Most of our analytical results will be derived for this special case. Section 3 analyzes equilibrium distribution of skills computed numerically for more than two states.

third stage, which is consumptions of goods produced in the good-shock and bad-shock sectors, is $c_H = y/\left(1+p^{1-\psi}\right)$ and $c_L = p^{-\psi}y/\left(1+p^{1-\psi}\right)$. This implies that the indirect utility is implying the indirect utility

$$V(y,p) = v(c_1(y,p), c_2(y,p)) = \left[\left(\frac{p^{-\psi}y}{1+p^{1-\psi}}\right)^{(\psi-1)/\psi} + \left(\frac{y}{1+p^{1-\psi}}\right)^{(\psi-1)/\psi}\right]^{\psi/(\psi-1)} = y. \quad (2)$$

Aggregating consumption across the consumers and using the resource constraints, we have $C_H = (pY_L + Y_H)/\left(1+p^{1-\psi}\right) = Y_H$ and $C_L = p^{-\psi}(pY_L + Y_H)/\left(1+p^{1-\psi}\right) = Y_L$, which implies

$$p = (Y_H/Y_L)^{\frac{1}{\psi}}. \quad (3)$$

The following proposition shows that although all workers are ex-ante identical, they will have different skill profiles ex-post. This result uses the fact that identical workers can take different actions as long as they are indifferent between outcomes.

Proposition 1 *(a) There will be always some individuals who fully specialize, that is, acquire only one sector skill. In addition, for A below some threshold $\bar{A} > 0$, (b) some individuals will choose to acquire both sector-specific skills, and (c) they will acquire them in equal amounts.*

Proof of Proposition 1. (c) This part of the proposition follows from strict concavity of the utility function. Notice that a worker acquires both skills only if he intents to always work in the highest wage sector, and therefore his wage per unit of effective labor is the same in both states, w_j. Then such a worker gets the highest expected utility by spending an equal amount of time on each skill, i.e., he acquires the two skills in equal amounts: $\arg\max_t\{\frac{1}{2}(w_j t^\gamma + \pi)^{1-\theta}/(1-\theta) + \frac{1}{2}[w_j(1-t)^\gamma + \pi]^{1-\theta}/(1-\theta)\} = \frac{1}{2}$.

(a) As we argued above, a worker acquires both skills only if he intents to always work in the better-paying sector. Therefore the only labor supply in the worse-paying sector comes from the workers who fully specialize in that sector skill. Since the marginal product of labor is infinite at zero, there must always be workers who fully specialize, and this will be true for both sectors.

(b) We prove this part by analyzing the threshold value $\bar{A}$ at which workers start to deviate from perfect specialization. This threshold value is characterized by indifference of any worker between staying perfectly specialized and deviating to $t = 1/2$, given that all other workers perfectly specialize, half working in one sector and half in the other. In this case the wage in the good-shock sector is $w_H = \alpha L_H^{\alpha-1} = \alpha[1/2]^{\alpha-1}$, and the wage in the bad-shock sector is $w_L = \alpha p\bar{A}L_L^{\alpha-1} = \alpha p\bar{A}(1/2)^{\alpha-1}$, so that using (3), $w_L/w_H = p\bar{A} = \bar{A}^{(\psi-1)/\psi}$. Notice that for $\psi = 1$, with all workers located at the edges, wages in the two sectors are exactly equal, and thus no worker will want to deviate from perfect specialization. When $\psi > 1$, we have $w_H > w_L$, so the sector with a good shock (call it the booming sector) also has higher wages compared to the bad-shock (stagnating) sector. In this case some workers will want to acquire both skills and always work in the good-shock sector. When $\psi < 1$, the sector with a good shock actually pays lower wage compared to the sector with a bad shock, that is, $w_H < w_L$. In this case some workers will again want to acquire both skills, but will always work in the bad-shock sector. Intuitively, when the goods are complements, the scarce good (the good produced in the bad-shock sector) becomes very desirable, and the economy will want more people producing it. On the contrary, when the goods are substitutes, the good produced in the less efficient sector will be (partially) substituted by the good produced in the more efficient sector, making more people work in the more efficient sector.

The indifference condition takes the following form:

$$\frac{1}{2}\frac{(w_H+\pi)^{1-\theta}}{1-\theta}+\frac{1}{2}\frac{(w_L+\pi)^{1-\theta}}{1-\theta}=\frac{\left(w_j\left[\frac{1}{2}\right]^{\gamma}+\pi\right)^{1-\theta}}{1-\theta}, \tag{4}$$

where $j = H$ for $\psi > 1$, and $j = L$ for $\psi < 1$. Using $L_H = L_L = \frac{1}{2}$, we can rewrite this as

$$\frac{1}{2}\frac{\left(1+\frac{1-\alpha}{\alpha}\left[\frac{1+\bar{B}}{2}\right]\right)^{1-\theta}}{1-\theta}+\frac{1}{2}\frac{\left(\bar{B}+\frac{1-\alpha}{\alpha}\left[\frac{1+\bar{B}}{2}\right]\right)^{1-\theta}}{1-\theta}=\frac{\left(\left[\frac{1}{2}\right]^{\gamma}+\frac{1-\alpha}{\alpha}\left[\frac{1+\bar{B}}{2}\right]\right)^{1-\theta}}{1-\theta}, \tag{5}$$

where $\bar{B}=\bar{A}^{\left|\frac{\psi-1}{\psi}\right|}$. Notice that $\bar{B} < 1$ creates a spread in the income of those who specialize. For $\bar{B}$ sufficiently high the left-hand side of the above equation exceeds the right-hand side, but when $\bar{B}$ decreases, we achieve an equality of the two sides. To see why $\bar{B} > 0$ (and therefore $\bar{A} > 0$), notice that with $\bar{B} = 0$ the mean incomes of specialists and generalists equal to $1/2 + (1-\alpha)/2\alpha$ and $(1/2)^{\gamma} + (1-\alpha)/2\alpha$, respectively. Since $\gamma < 1$, we have that $\bar{B} > 0$ (even for the linear utility). ∎

For the case of logarithmic utility ($\theta = 1$) we can find $\bar{B}$ from the above proof explicitly. The indifference condition can be written as

$$\left(1+\frac{1-\alpha}{\alpha}\left[\frac{1+\bar{B}}{2}\right]\right)\left(\bar{B}+\frac{1-\alpha}{\alpha}\left[\frac{1+\bar{B}}{2}\right]\right)=\left(\left[\frac{1}{2}\right]^{\gamma}+\frac{1-\alpha}{\alpha}\left[\frac{1+\bar{B}}{2}\right]\right)^{2}, \tag{6}$$

which after some algebra can be written as a quadratic equation in B:

$$\bar{B}^2+2\left(\frac{1}{1-\alpha}-\left[\frac{1}{2}\right]^{\gamma}\right)\bar{B}+\underbrace{1-2\left[\frac{1}{2}\right]^{\gamma}-\frac{2\alpha}{1-\alpha}\left[\frac{1}{2}\right]^{2\gamma}}_{\equiv c<0}=0. \tag{7}$$

Two roots of the above equation satisfy $\bar{B}_1\bar{B}_2 = c$. Since $c < 0$, one root is negative and the other is positive, this is the one we need.[4] We can then find the threshold level as $\bar{A}=\bar{B}^{|\psi/(\psi-1)|}$.

[4] We do not need to check that this root is below 1, since for $B > 1$ equation (6) (or its analog for $\psi < 1$) cannot hold at equality, since the left-hand side will always strictly exceed the right-hand side.

When $\gamma = 0$, the solution to equation (7) is $\bar{B} = 1$, so that $\bar{A} = 1$. Since $t^0 = 1$ for any $t > 0$, $\gamma = 0$ means that it is better to acquire both skills, and always work in the better-paying sector, even if there is no wage fluctuation, which gives us the threshold level of 1. When $\gamma > 0$, we get $\bar{A} < 1$ $\left(\bar{B} < 1\right)$. In addition, it can be shown that $\bar{A}$ is decreasing in γ. Intuitively, less decreasing returns in education make it more rewarding to specialize, and so a higher variation of wages would be needed to make workers to acquire both skills.

The derivation of $\bar{A}$ for the constant relative risk aversion (CRRA) utility function with $\theta \neq 1$ is much more complicated. However, it is intuitively clear that $\bar{A}(\theta)$ is an increasing function of θ, that is, higher risk aversion implies a higher threshold. That is, if a household dislikes risk more, a smaller wage variation can make him switch from specialization, which brings uncertainty in wages, to full insurance. In particular, it is easy to check that for a linear utility function the threshold level is $\bar{A}\left(\theta = 0\right) \equiv \hat{A} = \left(2\left[\frac{1}{2}\right]^{\gamma} - 1\right)^{|\psi/(\psi-1)|}$, which is strictly below $\bar{A}\left(\theta = 1\right) \equiv \bar{A}$ for all γ.

As an illustrative example, Figure 1 plots the thresholds $\bar{B}$ for logarithmic and linear utilities against γ. For $\psi = \infty$ these will be also the values of thresholds $\bar{A}$ for corresponding utilities.

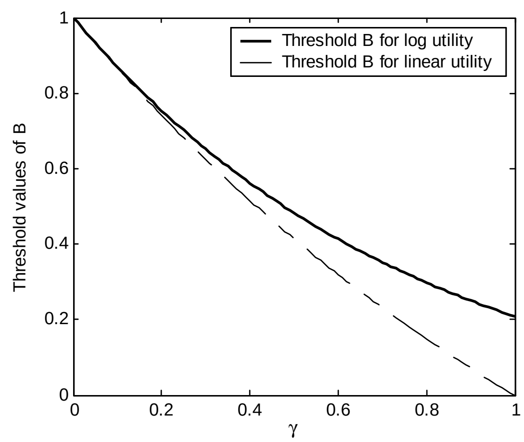

Figure 1. Threshold values $\bar{B}$ for logarithmic and linear utility.

Finally, ψ being further away from 1 implies a higher threshold. This immediately follows from the expression for $\bar{A} = \bar{B}^{|\psi/(\psi-1)|}$. A higher substitution between the goods results in a smaller increase in the price of the bad-shock good and thus in lower wages in the bad-shock sector, which encourages workers to move away from perfect specialization. We summarize our observations in the result below.

Claim 1 *a) The more risk averse the workers are, the higher the threshold* $\bar{A}$ *is, with* A *below which some workers choose to acquire both skills.*

b) The lower the γ (the more concave the education function), the higher the threshold $\bar{A}$ is.

c) The further the elasticity of substitution ψ from one, the higher the threshold $\bar{A}$ is.

As a result, we should expect less people to fully specialize in an economy with higher θ, lower γ, and higher ψ. The first part of this conjecture (comparative statics with respect to θ) will be formally proven in Claim 2 at the end of this subsection. (The other parts could be proven in a similar fashion.)

Now assume that A is given and $A < \bar{A}$. An indifference condition similar to (4) can be used to determine the degree of specialization in the economy. For each sector let δ be the fraction of workers who perfectly specialize in this sector's skill, and thus there remain $(1 - 2\delta)$ workers who acquire both skills.

For the remainder of the paper we will focus on the case of $\psi > 1$, so that all individuals who generalize, work in the good-shock sector. (The case with $\psi < 1$ will be symmetrical.) Then the effective labor supplied in the booming sector is $L_H(\delta) = \delta + (1/2)^{\gamma}(1 - 2\delta) = (1/2)^{\gamma} - \delta\left[2(1/2)^{\gamma} - 1\right]$, and the effective labor supplied in the stagnating sector is $L_L(\delta) = \delta$. This can be used to obtain the corresponding wages and profits:

$$w_H(\delta) = \alpha L_H^{\alpha-1}(\delta),\ w_L(\delta) = \alpha p A L_L^{\alpha-1}(\delta),\ \pi(\delta) = (1-\alpha)\left[L_H^{\alpha}(\delta) + pAL_L^{\alpha}(\delta)\right]. \tag{8}$$

Using the expression for indirect utility, (2), the utility of a worker who fully specializes (located at an edge of the unit interval) and of a worker who acquires both skills (located in the middle of the unit interval), as functions of δ, are $U_{edges}(\delta) \equiv \frac{1}{2}u(y_H(\delta)) + \frac{1}{2}u(y_L(\delta))$ and $U_{middle}(\delta) \equiv u(y_M(\delta))$, where

$$y_H(\delta) \equiv w_H(\delta) + \pi(\delta),\ y_L(\delta) \equiv w_L(\delta) + \pi(\delta),\ y_M(\delta) \equiv (1/2)^{\gamma} w_H(\delta) + \pi(\delta). \tag{9}$$

Lemma 1 *a) $dw_H/d\delta > 0$ and $dw_L/d\delta < 0$.*

b) $dU_{edges}/d\delta < dU_{middle}/d\delta$.

c) $dU_{edges}/d\delta < 0$.

d) For ψ sufficiently close to 1, $dU_{middle}/d\delta < 0$, and for ψ big enough ($\psi \geq 2$ is sufficient) $dU_{middle}/d\delta > 0$.

Part a) of Lemma 1 says that as fewer workers are located at the edges, the spread in income of specialists declines. Part b) says that as we move some workers from the edges to the middle, the utilities of those at the edges and of those in the middle become closer to each other. Part c) claims that the utility of workers located at the edges necessarily increases as some workers are moved away from the edges. Part d) claims that for ψ sufficiently big, the utility of workers located in the middle would decline. The proof of Lemma 1 is in the Appendix.

Recall that in the decentralized equilibrium all workers must have the same utility level. Therefore the decentralized equilibrium value of δ, denoted by δ^{DE}, is implicitly determined by the following indifference condition:

$$U_{edges}\left(\delta^{DE}\right) = U_{middle}\left(\delta^{DE}\right). \tag{10}$$

Lemma 1 and equation (10) imply the following comparative statics result.

Claim 2 *In the decentralized equilibrium, a higher risk aversion implies a lower proportion of workers who fully specialize in a particular skill, i.e., lower* δ^{DE}.

Proof of Claim 2. Suppose that for some θ equation (10) is solved by $\delta(\theta)$. Now consider $\theta' > \theta$. Since it is only specialists who suffer from income variation, with the old level of $\delta(\theta)$ we have that $U_{edges} < U_{middle}$. By Lemma 1, it order to bring the two utility levels back to equality, we need to decrease δ. Hence $\delta(\theta') < \delta(\theta)$. ■

To evaluate efficiency of the decentralized equilibrium, we want to study the allocation that would be chosen by a social planner. We devote the next subsection to this issue, and we find that the competitive equilibrium generates inefficiently little specialization.

2.2 The First-Best Allocation

We call the first-best (or unconstrained optimum) an allocation that maximizes the social welfare function[5] and in which ex-post (after the shocks are realized) transfers among the workers can be used. In this case there is full insurance, and the workers should be trained such that the aggregate expected output is maximized:

$$W^{FB}(\delta) \equiv 2\delta U_{edges} + (1-2\delta)U_{middle} = 2\delta\left[c_{edges,L}^{(\psi-1)/\psi} + c_{edges,H}^{(\psi-1)/\psi}\right]^{(1-\theta)\psi/(\psi-1)} / (1-\theta) +$$
$$(1-2\delta)\left[c_{middle,L}^{(\psi-1)/\psi} + c_{middle,H}^{(\psi-1)/\psi}\right]^{(1-\theta)\psi/(\psi-1)} / (1-\theta) \to \max_{c_{spec},\ c_{arb},\ \delta}$$

subject to $2\delta c_{edges,L} + (1-2\delta)c_{middle,L} = AL_L^\alpha$, $2\delta c_{edges,H} + (1-2\delta)c_{middle,H} = L_H^\alpha$, $L_L = \delta$, and $L_H = (1/2)^\gamma - \delta[2(1/2)^\gamma - 1]$. Denoting by λ_L and λ_H the Lagrange multipliers on the resource constraints and taking the first-order conditions for consumption, we obtain the optimal solution

[5] We assume that in the social welfare function all workers are given equal weights.

$c_{edges,j} = c_{middle,j}$, $j = L, H$. Then the first-order condition with respect to δ is

$$\frac{dW^{FB}}{d\delta} = \lambda_L \alpha A L_L^{\alpha-1} - \lambda_H L_H^{\alpha-1}\left[2\left[\frac{1}{2}\right]^\gamma - 1\right] \begin{cases} < 0, & \delta^{FB} = 0, \\ = 0, & \delta^{FB} \in \left(0, \frac{1}{2}\right), \\ > 0, & \delta^{FB} = \frac{1}{2}, \end{cases} \tag{11}$$

where δ^{FB} is the optimal choice of δ and $\lambda_L/\lambda_H = (Y_H/Y_L)^{1/\psi} = (L_H^\alpha / A L_L^\alpha)^{1/\psi}$ is the shadow price of good produced in the bad-shock sector. (Notice that it is equal to the market price p that would be determined in a competitive equilibrium.) Condition (11) can then be written as

$$A^{\frac{\psi-1}{\psi}}\left(\frac{\delta}{(1/2)^\gamma - \delta\left[2(1/2)^\gamma - 1\right]}\right)^{\alpha-1-\alpha/\psi} \lesseqgtr \left[2(1/2)^\gamma - 1\right]. \tag{12}$$

First, it is obvious that $\delta = 0$ is never optimal, since at this point the left-hand side of the above equation is infinite while the right-hand side is finite. In addition, we can see that if $\gamma = 1$, then the right-hand side of the above equation is zero implying full specialization at the optimum, i.e., $\delta^{FB} = 1/2$. Let us see under which conditions we get full specialization in the $\gamma < 1$ case. Evaluating (12) at $\delta = 1/2$, obtain $A \lesseqgtr \left[2(1/2)^\gamma - 1\right]^{\psi/(\psi-1)}$. Notice that the right-hand side of this expression is the threshold for the linear utility case, $\hat{A}$. According to our assumption $A < \bar{A}$ and part a) of Claim 1, we have that $A < \bar{A} > \hat{A} = \left[2(1/2)^\gamma - 1\right]^{\psi/(\psi-1)}$, where $\hat{A}$ is the threshold with linear utility. So if $A \geq \hat{A}$ (that is, A lies in between the solid and the dashed lines in Figure 1), then the optimal value of δ is $1/2$ (perfect specialization), while if $A < \hat{A}$ (i.e., A is below the dashed line), then $\delta^{FB} \in (0, 1/2)$, so the first best will involves locating some workers in the middle of the unit interval (less than perfect specialization).

The first-best allocation would be achieved, for example, if the workers had an access to a perfect contingent claims market or if they were risk-neutral. This observation and Claim 2 imply the following result for risk-averse workers:

Proposition 2 *The first best involves more specialization than the decentralized equilibrium.*

Having obtained this result, it is interesting to see whether a planner who cannot use transfers among the workers sets the same level of specialization as the decentralized equilibrium. The next subsection addresses this question.

2.3 Constrained Optimum Allocation

We define the constrained optimum in the following way. Suppose the social planner chooses how much time each worker should spend on each skill, but he cannot transfer consumption among the workers. His optimization problem can be written as

$$\begin{aligned} W^{CO}(\delta) &\equiv 2\delta U_{edges}(\delta) + (1-2\delta)U_{middle}(\delta) \\ &= 2\delta\left[\frac{1}{2}u\left(y_H(\delta)\right) + \frac{1}{2}u\left(y_L(\delta)\right)\right] + (1-2\delta)u\left(y_M(\delta)\right) \ \rightarrow \ \max_{\delta} \end{aligned}$$

subject to (8) and (9). That is, we still assume that the workers must receive their marginal products of labor, and the profits are distributed equally among the workers. The first-order condition for the above problem is

$$\frac{dW^{CO}}{d\delta} = 2\left(U_{edges} - U_{middle}\right) + \left[2\delta\frac{dU_{edges}}{d\delta} + (1-2\delta)\frac{dU_{middle}}{d\delta}\right] \begin{cases} < 0, & \delta^{CO} = 0, \\ = 0, & \delta^{CO} \in \left(0, \frac{1}{2}\right), \\ > 0, & \delta^{CO} = \frac{1}{2}, \end{cases} \tag{13}$$

where δ^{CO} is the constrained optimal choice of δ. The marginal effect from moving one worker from each edge to the middle can be decomposed into two terms. First, we give these two workers U_{middle} instead of U_{edges}, which is reflected in the first term of equation (13). Second, there is a marginal change in labor supplies by these two workers, which affects equilibrium prices, and hence the utilities of all workers. This is reflected in the second term of condition (13).

Lemma 2 *If utility is logarithmic ($\theta = 0$), then*

$$2\delta\frac{dU_{edges}}{d\delta} + (1-2\delta)\frac{dU_{middle}}{d\delta}\bigg|_{\delta=\delta^{DE}} < 0. \tag{14}$$

Although we have the analytical proof only for the case of $\theta = 0$, in numerical computations this result holds for $\theta \neq 0$ as well. The proof of Lemma 2 can be found in the Appendix. The above lemma implies the following result.

Corollary 1 *If utility is logarithmic ($\theta = 0$), then*

$$\frac{dW^{CO}}{d\delta}\bigg|_{\delta=\delta^{DE}} < 0. \tag{15}$$

This result follows from the fact that in the decentralized equilibrium $U_{edges} = U_{middle}$, so that by Lemma 2, the equation (13) implies the above inequality. Inequality (15) means that at $\delta = \delta^{DE}$ the planner is (locally) better off by moving to $\delta < \delta^{DE}$. In words, the cut in the utility of workers in the middle is worth it because it is offset by the gain in the utility of the workers at the edges.

For ψ sufficiently close to 1, we have $dU_{edges}/d\delta < dU_{middle}/d\delta < 0$, and thus condition (14) will also hold for $\delta \geq \delta^{DE}$ (since for $\delta \geq \delta^{DE}$ we have that $U_{edges} \leq U_{middle}$, and thus the first-order condition (13) would be satisfied). To show that the global maximum must be located to the left of δ^{DE} for any ψ, a sufficient condition would be concavity of the welfare function. An even stronger sufficient condition would be equation (14) holding for $\delta \geq \delta^{DE}$ (since for $\delta \geq \delta^{DE}$ we have that $U_{edges} \leq U_{middle}$, and thus the first-order condition (13) would be satisfied). We know that at $\delta = \frac{1}{2}$, $2\delta dU_{edges}/d\delta + (1-2\delta)dU_{middle}/d\delta$ is strictly negative by Lemma 1. Lemma 2 shows that it is also strictly negative at $\delta = \delta^{DE}$. If it is monotone in between, we have that it is negative for all $\delta \geq \delta^{DE}$. However, we did not succeed to prove this conjecture or concavity of the welfare function. Nevertheless, even for $\psi = \infty$, all of many combinations of model parameters that we have tried in numerical computations, gave us a concave welfare function as well as $2\delta dU_{edges}/d\delta + (1-2\delta)dU_{middle}/d\delta|_{\delta \geq \delta^{DE}} < 0$ holding. Below are two representative figures that reflect this fact. We used the following parameter values: $\psi = \infty$ (which is the case for which $dU_{middle}/d\delta$ is the highest), $\theta = 1.25$, $\alpha = .75$, $\gamma = .7$, $A = .3$.

Figure 2 shows $U_{edges}(\delta)$, $U_{middle}(\delta)$, and $W^{CO}(\delta) \equiv 2\delta U_{edges}(\delta) + (1-2\delta)U_{middle}(\delta)$ plotted against δ. The circle denotes the point of the decentralized equilibrium, where $U_{edges} = U_{middle}$. The asterisk denotes a point at which the welfare is maximized. We can see that the asterisk is to the left of the circle, that is, $\delta^{CO} < \delta^{DE}$. Also, the welfare function is concave.

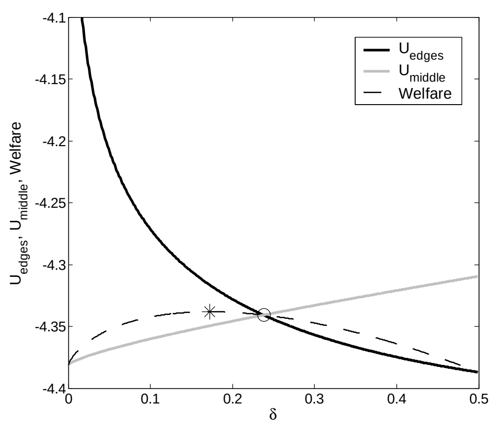

Figure 2. $U_{edges}\,(\delta)$, $U_{middle}\,(\delta)$, and $W^{CO}\,(\delta)$. $\delta^{SB}\approx .17$, $\delta^{CE}\approx .24$.

The left panel of Figure 3 plots $U_{edges}/d\delta$, $dU_{middle}/d\delta$, and $2\delta dU_{edges}/d\delta + (1-2\delta)dU_{middle}/d\delta$ against δ. The maximum welfare and the decentralized equilibrium are again denoted by an asterisk and a circle. You can see that the dotted curve (that corresponds to $2\delta dU_{edges}/d\delta + (1-2\delta)dU_{middle}/d\delta$) is below zero for all points to the right of the circle, and it is also monotone. The right panel of Figure 3 plots $dW^{CO}/d\delta$ as a function of δ. Not surprisingly, it is equal to zero at the point of the maximum welfare, and is negative at δ^{DE}. We summarize our findings in the result below.

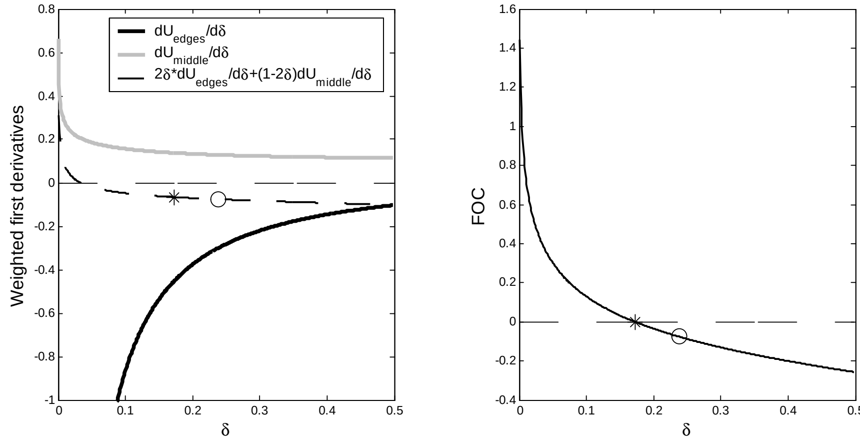

Figure 3. Left panel: $U_{edges}/d\delta$, $dU_{middle}/d\delta$, and $2\delta dU_{edges}/d\delta + (1-2\delta)dU_{middle}/d\delta$.
Right panel: $dW^{CO}(\delta)/d\delta$.

Numerical result *The constrained optimum (where all the workers are weighted equally in the welfare function) involves less specialization than the decentralized equilibrium. In the constrained optimum the specialists receive higher utility than those workers who acquire both skills.*

Combining this result with Propositions 2, we have the following relationship for the fractions of specialized workers and for the social welfare: $\delta^{FB} > \delta^{DE} > \delta^{CO}$ and $W^{FB} > W^{CO} > W^{DE}$. In words, the first-best allocation involves the most specialization, the constrained optimum involves the least of it, and the decentralized equilibrium is in between. Obviously, the first best delivers the highest welfare, then comes the constrained optimum, and then the decentralized equilibrium.

However, whether the constrained optimum is Pareto improving relative to the decentralized equilibrium depends on the value of ψ. Only for ψ very close (but not equal[6]) to 1, both specialists and generalists benefit from a decrease in the fraction of workers located at the edges. (The reason is that both $dU_{edges}/d\delta$ and $dU_{middle}/d\delta$ are strictly negative, so that both black and grey curves on Figure 2 are downward sloping.)[7] However, for sufficiently high ψ (definitely for $\psi \geq 2$, but this is a very strong sufficient condition, see the proof of Lemma 1) we have $dU_{middle}/d\delta > 0$, and thus even though at the constrained optimum workers at the edges are made better off compared to the decentralized equilibrium, workers in the middle are made worse off.

[6]Recall that for $\psi = 1$ there is no variation in wages, so both competitive equilibrium and unconstrained optimum achieve the first best.

[7]Even though w_H falls as the number of workers in the middle rises, this fall is compensated by an increase in profits (which equal to a constant fraction of the value of aggregate output in the two sectors). Thus the total income of those who generalize rises, making them better off. In the decentralized equilibrium this general equilibrium effect is not internalized by the workers, which results in a less than efficient (given the no-transfers restriction) fraction of workers located in the middle.

Intuitively, the constrained planner can improve social welfare compared to the competitive equilibrium (though not Pareto improve for most values of ψ), because in the decentralized equilibrium workers only internalize the fact that by specializing they increase variation in their own income, but the increase in variation in the income of others is external to them. Murphy (1986) uses this intuition to conjecture that the planner can improve upon competitive equilibrium, even though neither does he prove it analytically, nor does he investigate whether this improvement can be Pareto. In fact, the improvement can be Pareto only because the profits are redistributed back to the workers, which Murphy (1986) does not model.

As we argued before, for most values of ψ (so that $dU_{middle}/d\delta > 0$) the improvement is not Pareto. In fact, in this case we can find such Pareto weights for which the solution to the constrained planner's problem will coincide with the decentralized equilibrium allocation. Notice that as long as the weights that the planner assigns to workers sum up to one, the welfare function goes through the point of intersection of $U_{edges}(\delta)$ and $U_{middle}(\delta)$. (See Figure 2.) Thus we need to find such weights that this point of intersection is where the welfare achieves its maximum. This means that $U_{edges}(\delta)$ must be weighted less, or, another words, the planner will put workers with lower Pareto weights to the edges. In such an optimum all workers will receive the same utility, even though they have different Pareto weights. It also seems rather ad-hoc to weight ex-ante identical workers differently. On the other hand, the result that at the optimum where all workers have equal weights some workers nevertheless receive higher utility than others is somewhat paradoxical as well.

3 Numerical Computation of Decentralized Equilibrium

All the analytical results derived in the previous section were obtained for a simple case of the shocks distribution given by (1). We saw that in this case the equilibrium skill distribution has only three points of strictly positive density – the two edges and the middle. It is natural to wonder how the equilibrium distributions would look like for more general distributions of shocks. In this section we present numerical solutions for two examples: 1) uniform and perfectly correlated shocks (similar to the one given by (1), but with more than two possible shock values, and 2) uniform and independent shocks. We will also discuss differences and similarities of the resulting equilibrium skill distributions. For all examples below we used the following parameter values:[8] $\theta = 1.25$, $\alpha = .75$, $\gamma = .7$. We consider each sector shock z_i, $i = 1, 2$, to be uniformly distributed

[8] Higher θ, higher α, and lower γ result in less specialization (put more people away from the edges).

over an equally-spaced vector $(a_1, ..., a_n)$ with $a_n = 1$.

Example 1. Uniform and Perfectly Negatively Correlated Productivity Shocks

In this example we assume that shocks in the two sectors are perfectly negatively correlated such that whenever $z_1 = a_j$, we have $z_2 = a_{n+1-j}$. In addition, all of the pairs $(z_1, z_2) = (a_j, a_{n+1-j})$ are equally probable. In this example we use $n = 30$ and $a_1 = .01$. Figure 5 shows the equilibrium distribution and the corresponding expected utility for $\psi = \infty$ (i.e., the two goods are perfect substitutes). The horizontal axis is the unit interval that can be viewed as time spent on sector 2 skill. A worker located at zero fully specializes in sector 1 skill, and a worker located at one fully specializes at sector 2 skill.

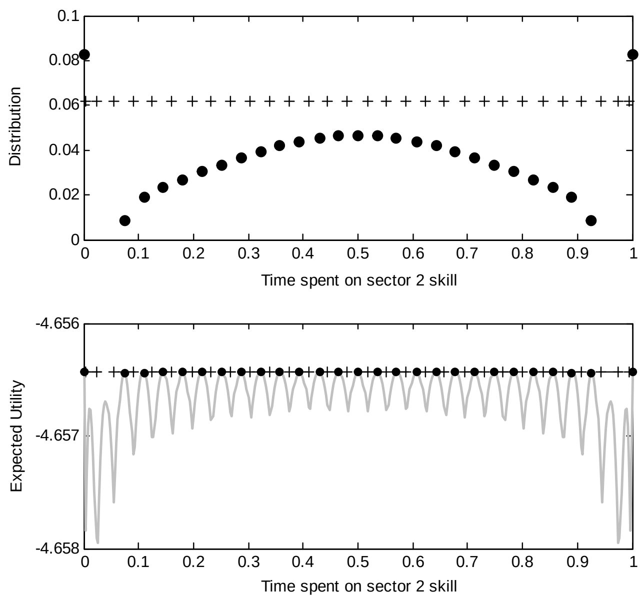

Figure 4. Equilibrium distribution and expected utility for uniform perfectly negatively correlated productivity shocks. $\psi = \infty$, $n = 30$, a_1= .01.

At the top panel of Figure 4 the black circles denote the mass points of the distribution. The crosses have the following meaning. Given the distribution of workers, for each pair of shocks (z_1, z_2) we find a worker who is just indifferent between going to sector 1 and sector 2 (for that particular pair of shocks), given that all workers to the left of him go to sector 1, and all workers to the right of him go to sector 2. The crosses on the figure denote these marginal locations for each pair (z_1, z_2), the order of the crosses from the left to the right corresponds to the ratio $\frac{z_1}{z_2}$ being in ascending order. Notice that all workers in between each two adjacent crosses have exactly the same equilibrium strategy (they travel to a particular sector under the same conditions), and therefore they must be located at the same point. In other words, there can be at most one mass point in between each pair of adjacent crosses. This observation is also reflected on the bottom panel of Figure 4, where the grey line denotes the expected utility as a function of location. On each interval in between two adjacent crosses, expected utility has a single local maximum. The black circles on the bottom panel correspond to the expected utility for points where the distribution density is strictly positive. We can see that all these levels are equal – all lie on the dashed line that denote their mean value – and the grey line lies below this level.

We can see that for perfectly correlated productivity shocks there are some workers at the edges and some workers around the middle of the interval. Notice also that there never will be workers right next to the edges, in particular, to the left of the first cross and to the right of the last cross. The reason is that if there were people located at the interval from zero to the first cross (from the last cross to one), they would travel to sector 1 (sector 2) for any pair of the shocks, and therefore they would have been better off by acquiring that sector skill only. This is very intuitive – it does not make sense to "almost" fully specialize, because the amount of the other skill you acquire is so little that you never find it profitable to work in the other sector, and thus it is not worth it to acquire that skill in the first place.[9]

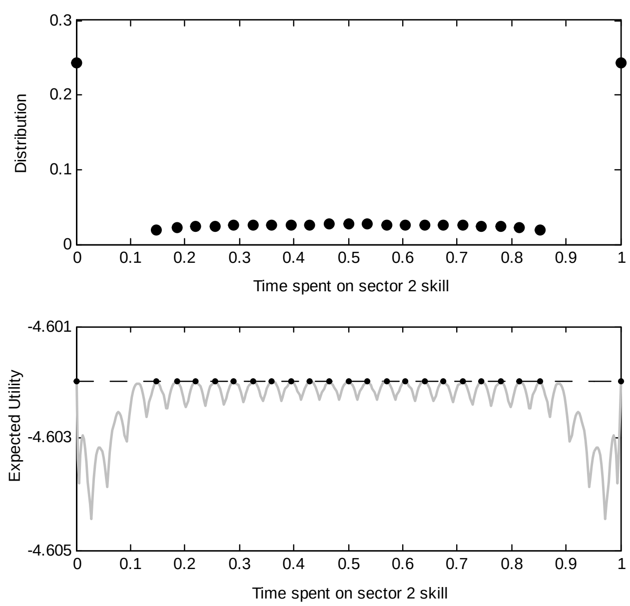

Figure 5. Equilibrium distribution and expected utility for uniform perfectly negatively correlated productivity shocks. $\psi = 10$, $n = 30$, $a_1 = .01$.

Figure 5 shows an analog of Figure 4 for $\psi = 10$. Again, there are mass points right at the edges, no workers next to the edges, and then a "hump" of mass points around the middle of the

[9]In fact, this result holds for very concave education production function (γ close zero) as well.

interval. We will see in Example 2 that this kind of shape is particular to the perfectly correlated shocks case, and the figures look very different once we drop this assumption. In addition, the $\psi = 10$ results in less generalization (a smaller hump around the middle) compared to the $\psi = \infty$ case. This is consistent with our theoretical prediction that the further the elasticity of substitution from one, the more important it is to have generalized workers so that they switch to production of the good in the good-shock (if $\psi > 1$, and bad-shock if $\psi < 1$) sector. Figure 6 corresponds to the case of $\psi = .35$. There is no hump, instead it is somewhat spread towards the edges, but still there are workers around the middle of the interval.

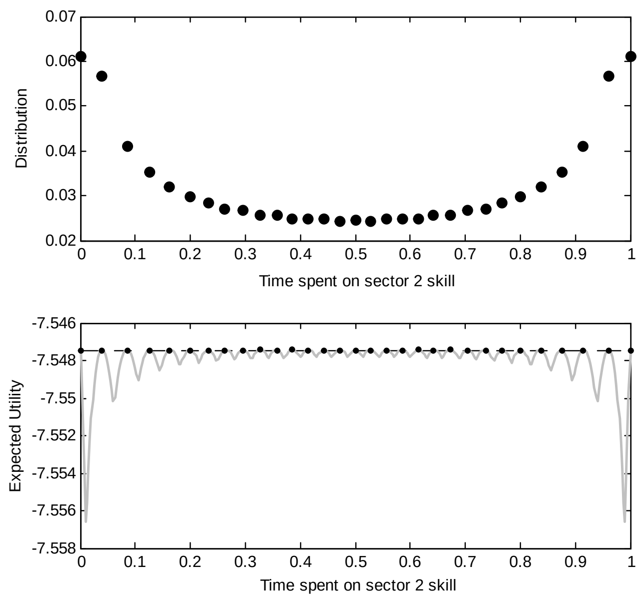

Figure 6. Equilibrium distribution and expected utility for uniform perfectly negatively correlated productivity shocks. $\psi = .35$, $n = 30$, $a_1 = .01$.

Remember that with $\psi < 1$ workers travel to the bad-shock sector, since when the goods are complements, it is important to produce both of them. However, the shape of the distribution that we see on Figure 6 is not so much specific to the case of $\psi < 1$, but rather to how much ψ is different form 1 (recall that for $\psi = 1$ in equilibrium all workers will be located at the edges). In particular, you can see it on Figure 7, where we depict equilibrium distributions for $n = 2$, $a_1 = .3$, and three cases, $\psi = \infty$, $\psi = 10$, and $\psi = .35$. This is a case of the simple distribution for which we derive our analytical results in Section 2. We can see that for $\psi = \infty$ (black circles) there is the most workers in the middle, for $\psi = .35$ (white circles) there is the least workers in the middle, and the case $\psi = 10$ (grey circles) is in between.

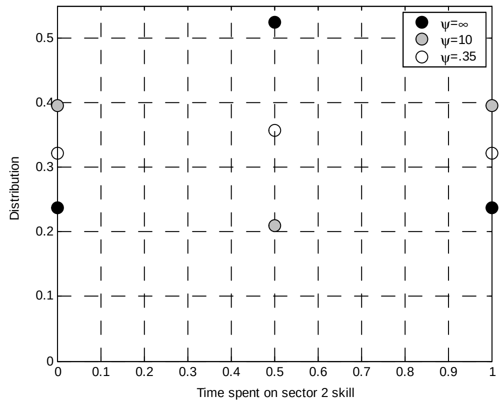

Figure 7. Equilibrium distributions for uniform perfectly negatively correlated productivity shocks for three cases, $\psi = \infty$, $\psi = 10$, and $\psi = .35$. Here $n = 2$, a_1= .3.

We now move to the example of uniform and independent productivity shocks. We will see that the shape of the skill distributions is very different from what we have seen in Example 1.

Example 2. Uniform i.i.d. Productivity Shocks

In this example we assume that shocks in the two sectors are independent, so that all pairs $(z_1, z_2) = (a_j, a_k)$, $j, k = 1, ..., n$, are assigned equal probabilities. We use $n = 15$ and $a_1 = .01$ in the three figures below. Figure 8 plots the equilibrium distribution and expected utility for the perfect substitutes case, $\psi = \infty$. You can see how again there are mass points at the edges, then no people next to the edges. But this is the end of the similarities between this example and the previous one. Now we have absolutely no workers over an interval around the middle, and some workers on the sides. Also notice that the distribution is rather chaotic, and this is not just an approximation error. In fact, there is no reason why the distribution should have some monotone shape. Think for example about three crosses next to each other. In the previous example $\frac{z_1}{z_2}$ being less than $\frac{z_1'}{z_2'}$ automatically implied that $z_1 < z_1'$ and $z_2 > z_2'$. This is not the case when the shocks are i.i.d., so we lose that sort of monotonicity moving from one interval between two adjacent crosses to another.

Figures 9 and 10 are analogs of Figure 8 for $\psi = 10$ and $\psi = .35$, respectively. One can see that the shapes of the distributions are similar to the one on Figure 8. However, $\psi = 10$ results in more people at the edges compared to $\psi = \infty$ and $\psi = .35$. For $\psi = .35$ we also have more people right at the edges compared to $\psi = \infty$, but then the distribution is more spread towards the middle of the interval.

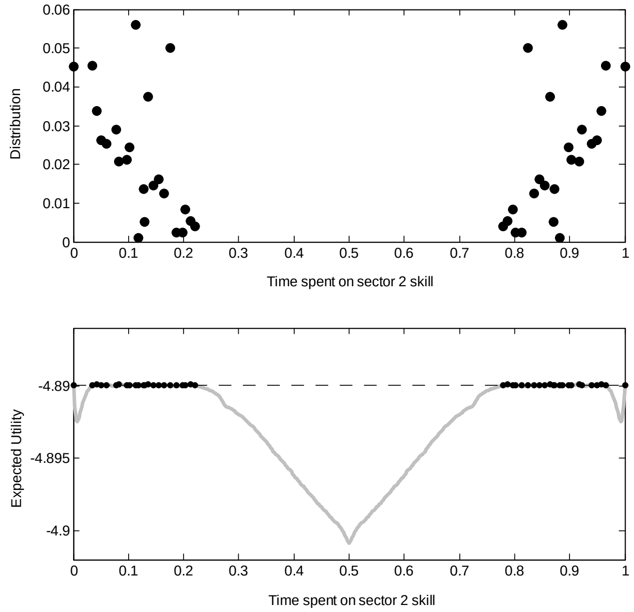

Figure 8. Equilibrium distribution and expected utility for uniform i.i.d. shocks. $\psi = \infty$, $n = 15$, a_1= .01.

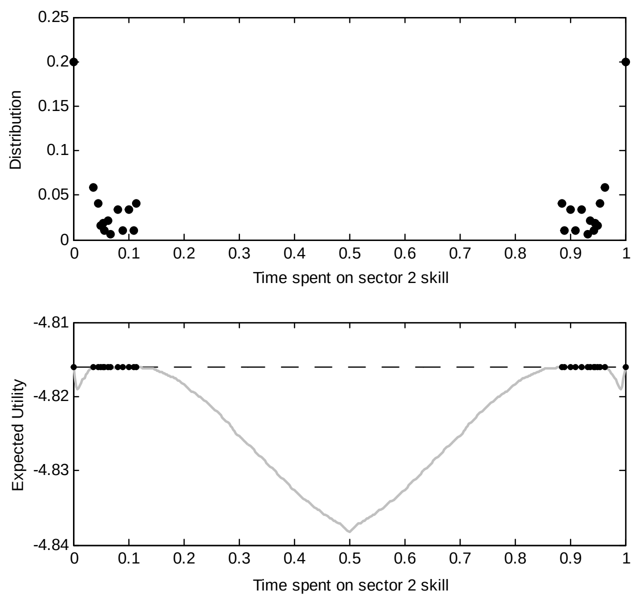

Figure 9. Equilibrium distribution and expected utility for uniform i.i.d. shocks. $\psi = 10$, $n = 15$, a_1= .01.

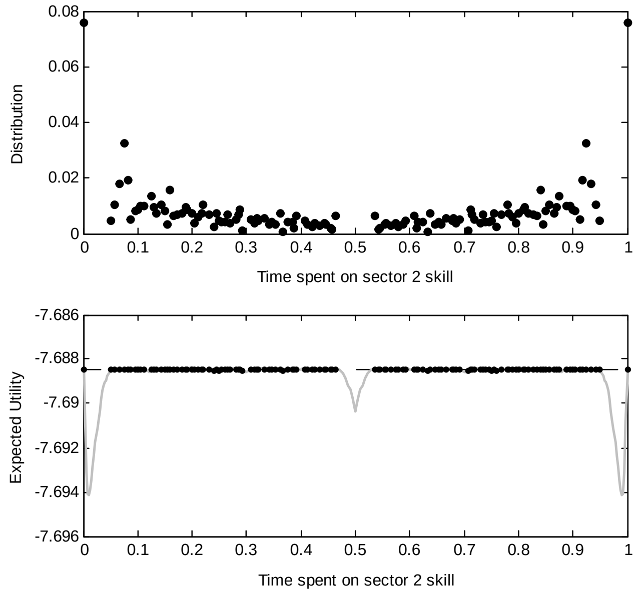

Figure 10. Equilibrium distribution and expected utility for uniform i.i.d. shocks. $\psi = .35$, $n = 15$, $a_1 = .01$.

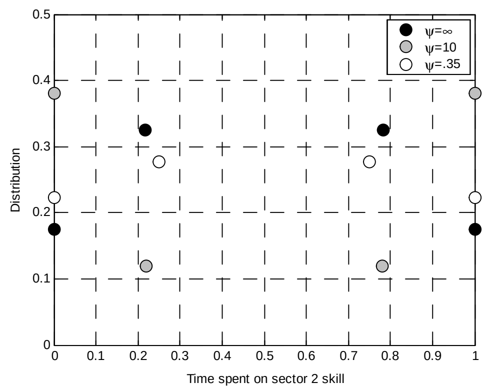

Figure 11. Equilibrium distributions for uniform i.i.d. shocks for three cases, $\psi = \infty$, $\psi = 10$, and $\psi = .35$. Here $n = 2$, $a_1 = .13$.

Figure 11 plots equilibrium skill distributions for $n = 2$, $a_1 = .13$, and three cases, $\psi = \infty$, $\psi = 10$, and $\psi = .35$. We can see that for $\psi = .35$ (white circles) the intermediate mass points are located closer to the middle of the interval than those for $\psi = \infty$ (black circles) and $\psi = 10$ (grey circles). In addition, we again obtain that $\psi = 10$ has the most workers at the edges, $\psi = \infty$ has the least workers at the edges, and $\psi = .35$ is in between. Also, notice similarities between Figures 7 and 11. The amount of workers at the edges are ordered in the same way, with $\psi = 10$ having the most of them, $\psi = \infty$ having the least, and $\psi = .35$ is in between.

In order to understand why with i.i.d. shocks there are no people around the middle of the interval, while perfectly correlated shocks it is the opposite, let us return to our simple example where the shock in each sector can take two values, 1 and $A < 1$. (The resulting distributions for the two examples are shown on Figures 7 and 11). We saw that with perfectly correlated shocks the workers who acquire both skills optimally choose to be exactly in the middle (see Figure 7), because they always want to work in the better-paying sector. In other words, they travel to sector 1 with probability 1/2 and to sector 2 with probability 1/2. With independent shocks, a worker who acquired both skills will want to travel to another sector only if that sector pays a strictly higher wage, which happens only with probability 1/4. In other words, a worker with both skills will want to locate himself somewhere in between an edge and the middle of the interval, and travel to the closest sector 3/4 of the time and to the further sector 1/4 of the time (more precisely, with probabilities 3/4 and 1/4). Thus in this simple example with two i.i.d. shocks in each sector, the equilibrium distribution will have four mass points – two at the edges, and two somewhere in between each edge and the middle, which is exactly what we see on Figure 11.

To summarize, what drives (part of the) workers away from perfect specialization in this model is the need of insurance and the concavity of the education function. With perfectly correlated shocks there is also an additional force. It comes from the fact that one sector is always better than the other, and so if one sector received a bad shock, it is necessarily true that the other sector received a good shock. This makes it appealing to a worker to locate himself in the middle, and always travel to the better-paying sector (with more than two shocks, instead we will have mass points concentrated around the middle of the interval). With the i.i.d. shocks, some workers still move away from perfect specialization, but not too far from each sector, so that they have to go to the other sector only when their own sector is strictly worse.

4 Endogenizing Capital Supply

In the model that we considered so far, profits were equally distributed among workers. In other words, all agents have the same portfolio of the firms' shares, or, equivalently, supply capital equally to the two sectors. In this section we consider two modifications of this setup.

First, we consider ex-ante capital supply choice. In other words, we suppose that before the productivity shocks are realized, workers can choose in which sector to supply capital that they own (or, equivalently, shares of which firm to buy). We show that by investing in the sector opposite from the one they acquire skill in, workers can partially eliminate the insurance problem, but not completely as long as labor's share is above 1/2, and hence there will be still less than perfect specialization in such an economy.

Second, we consider ex-post capital supply choice, i.e., when capital supply decisions can be made after the productivity shocks are realized. We show that in this case the problem of variation in wages in each sector is aggravated, so that an even smaller variation in shocks is needed for less than perfect specialization than in the original model (without capital mobility). Intuitively, capital flows into a sector where the return is higher, which decreases the wage in the worse-paying sector even further, making it less attractive to be a specialist.

4.1 Ex-Ante Capital Supply Choice

First, suppose that prior to shocks realization the workers can choose where to supply their capital. This can be used as an insurance device – the workers who specialize in sector 1 will supply capital to sector 2, and the other way around. The workers who acquire both skills (as before, we assume the simple distribution given by (1)) will supply their capital equally to two sectors (because ex-ante the two sectors look equally profitable). To be consistent with the setup of Section 2 (see footnote 1), assume that each household owns two units of capital. Then in equilibrium each sector will have one unit of capital. Assume that the production function is Cobb-Douglas with α being labor's share ($0 < \alpha < 1$). We can then prove an analog of Proposition 1. In particular, the threshold level $\bar{A}$ can be found from the indifference condition:

$$\frac{1}{2}\frac{(w_H + \pi_L)^{1-\theta}}{1-\theta} + \frac{1}{2}\frac{(w_L + \pi_H)^{1-\theta}}{1-\theta} = \frac{([1/2]^{\gamma}\, w_H + \pi_M)^{1-\theta}}{1-\theta}, \tag{16}$$

where $\pi_M = (1-\alpha)\,(1/2)^{\alpha}\left(1 + \bar{A}^{|(\psi-1)/\psi|}\right)$, $\pi_H = 2(1-\alpha)\,(1/2)^{\alpha}$, and $\pi_L = 2(1-\alpha)\left(\frac{1}{2}\right)^{\alpha}\bar{A}^{|(\psi-1)/\psi|}$. Equation (16), divided by w_H^2, becomes an equation in $\bar{B} = \bar{A}^{|(\psi-1)/\psi|}$:

$$\frac{1}{2}\frac{\left(1 + \frac{1-\alpha}{\alpha}\bar{B}\right)^{1-\theta}}{1-\theta} + \frac{1}{2}\frac{\left(\bar{B} + \frac{1-\alpha}{\alpha}\right)^{1-\theta}}{1-\theta} = \frac{\left([1/2]^{\gamma} + \frac{1-\alpha}{\alpha}\left[\frac{1+\bar{B}}{2}\right]\right)^{1-\theta}}{1-\theta}. \tag{17}$$

Compare the above condition with equation (5).The right-hand sides of equations (17) and (5) are exactly the same. The left-hand side of (17) is larger than the left-hand side of (5), because capital serves as partial insurance. Notice that the expected income of specialists is the same in both cases, which implies that for the linear utility case ($\theta = 0$), we get the same threshold, namely, $\bar{B} = 2\,(1/2)^{\gamma} - 1$.

When $\alpha = 1/2$, we have full insurance and $\bar{B} = 2\,(1/2)^{\gamma} - 1$, which is the threshold under the first best. If $\alpha < 1/2$, the workers will not supply all their capital to the opposite sector, but only a part of it, and will also achieve full insurance. However, as long as $\alpha > 1/2$, we have that

$w_H + \pi_L > w_L + \pi_H$, and thus the insurance is only partial. As a result, the threshold will be above the one under the first best, but below the one without insurance through capital. Intuitively, when workers can choose in which firm to invest, they eliminate the insurance problem partially, but not completely as long as labor income is a substantial part of their earnings.

To summarize, even if the workers can make an ex-ante choice in which sector to supply their capital, for labor's share big enough (and the education function concave enough and shocks variation high enough) there are still going to be workers who acquire both sector skills. Therefore for this case we can still apply the analysis similar to the one in Sections 2 and 3 to study a degree of specialization in such an economy. Notice also that we have considered a case of perfectly negatively correlated shocks, which is the strongest in terms of providing insurance by investing into the sector different from the one you specialize in. If the shocks are less than perfectly correlated, the insurance result will be weakened.

4.2 Ex-Post Capital Supply Choice

The setup of the previous subsection assumed that the capital supply choice was made prior to the shocks realization. It is interesting to look at the situation when capital can flow from one sector to another after the shocks have realized. If we view the skill acquirement decision as a life-time choice of profession, then it is quite natural to assume that the capital investment decision is relatively short-term. In this case, capital will flow from one sector to another until the marginal returns are equalized. Notice that this implies that as long as all households own the same amount of capital, their capital earnings will be the same, so capital income does no longer serve the insurance purpose, as we had in the previous subsection.

Again, assume that the goods production technology in each sector is Cobb-Douglas, $Y_j = z_j L_j^{\alpha} K_j^{1-\alpha}$, $j = 1, 2$, and that each household owns two units of capital. As before, consider a simple case with the shocks distribution given by equation (1). The rental rates of capital in the two sectors are $R_H = (1-\alpha) L_H^{\alpha} K_H^{-\alpha}$ and $R_L = (1-\alpha) \left[L_H^{\alpha} K_H^{1-\alpha} / \left(A L_L^{\alpha} K_L^{1-\alpha}\right)\right]^{1/\psi} A L_L^{\alpha} K_L^{-\alpha}$. These rates must be equalizes in equilibrium, implying

$$K_L/K_H = \left[L_H^{\alpha} K_H^{1-\alpha} / \left(A L_L^{\alpha} K_L^{1-\alpha}\right)\right]^{(1-\psi)/\psi}, \tag{18}$$

which can also be written as

$$K_L/K_H = \left(A L_L^{\alpha} / L_H^{\alpha}\right)^{(\psi-1)/[1+\alpha(\psi-1)]}. \tag{19}$$

The equilibrium wages are given by $w_L = \alpha \left[L_H^{\alpha} K_H^{1-\alpha} / \left(A L_L^{\alpha} K_L^{1-\alpha}\right)\right]^{1/\psi} A L_L^{\alpha-1} K_L^{1-\alpha}$ and $w_H = \alpha L_H^{\alpha-1} K_H^{1-\alpha}$, so that $w_L/w_H = \left[L_H^{\alpha} K_H^{1-\alpha} / \left(A L_L^{\alpha} K_L^{1-\alpha}\right)\right]^{(1-\psi)/\psi} L_H/L_L = (K_L/K_H)(L_H/L_L)$, where the last equality follows from (18).When all workers are located at the edges, so that $L_H = L_L = 1/2$, we have $w_L/w_H = K_L/K_H = A^{(\psi-1)/[1+\alpha(\psi-1)]}$ (where the last inequality follows from (19)) and $\pi/w_H = \left[1 + A^{(\psi-1)/[1+\alpha(\psi-1)]}\right](1-\alpha)/2\alpha$. Hence the indifference condition determining the threshold level is the same as (5), but with $\bar{B} = \left[\bar{A}'\right]^{(\psi-1)/[1+\alpha(\psi-1)]}$ instead of $\bar{B} = \bar{A}^{\frac{\psi-1}{\psi}}$, so that $\bar{A}' = \bar{A}^{[1+\alpha(\psi-1)]/\psi}$. Since for $\psi > 1$ we have $1+\alpha(\psi-1) < \psi$, it follows that $\bar{A}' > \bar{A}$, where $\bar{A}$ is the threshold level for the case with immobile capital studied in Proposition 1 of Section 2. That is, in the case with ex-post capital mobility a lower variation in shocks is needed for less than perfect specialization, than in the case without capital mobility. Intuitively, capital flows into a more productive (good-shock) sector which decreases the wage in the bad-shock sector, so that the income variation of a specialist increases, making it less attractive to be one.

The following special case is interesting. If the goods are perfect substitutes, i.e., $\psi = \infty$, then for any L_H and L_L the ratio of wages is constant: $w_L/w_H = A^{1/\alpha}$. As long as $A < 1$ (that is, there is at least some uncertainty), it is an equilibrium for all workers to be located in the middle of the interval (all working in the booming sector), and for all capital to be located in the booming sector. All output in the economy is then produced by the booming sector, and the wage and the rental rate of capital in the stagnating sector are zero. In this case a (risk-averse) worker located in the middle has no incentives to deviate to the edge of the interval, since in the first case his certain income is $(1/2)^\gamma w_H + \pi$, and in the second case even the mean of his income, $\frac{1}{2}w_H + \pi$, is lower, plus he faces uncertainty (he earns $0 + \pi$ with probability $1/2$ and $w_H + \pi$ with probability $1/2$).

There can be another equilibrium in this economy, in particular, the one in which workers are located both at the edges and in the middle, and capital is distributed among the two sectors so that the marginal products are equalized. Notice that the threshold level in this case equals $\bar{A}' = \bar{A}^\alpha$.

We consider the following three cases to characterize all possible equilibria in this economy.

Case 1: $A \geq \bar{A}'$. In this case there are two equilibria, one in which all workers are located in the middle, and the other in which all workers are located at the edges.

Now suppose that $A < \bar{A}'$. We have that for $\delta = 1/2$ (i.e., when all workers are located at the edges),

$$(1 + \pi/w_H)\left(A^{1/\alpha} + \pi/w_H\right) < \left([1/2]^\gamma + \pi/w_H\right)^2. \qquad (20)$$

The expression for π/w_H, for given L_L and L_H, is

$$\pi/w_H = 2R_H/w_H = 2\frac{(1-\alpha)}{\alpha}L_H/K_H. \qquad (21)$$

Using the condition of equal capital returns in the two sectors, $L_H/K_H = A^{1/\alpha}L_L/K_L = A^{1/\alpha}L_L/(2-K_H)$, obtain $K_H = 2L_H/\left(A^{1/\alpha}L_L + L_H\right)$. Plugging this equation into equation (21), we have $\frac{\pi}{w_H} = \left[A^{1/\alpha}L_L + L_H\right](1-\alpha)/\alpha$. Let us see how π/w_H changes as δ decreases. Using $L_L = \delta$ and $L_H = (1/2)^{\gamma} - \delta\left[2(1/2)^{\gamma} - 1\right]$, obtain $d(\pi/w_H)/d\delta = \{A^{1/\alpha} - [2(1/2)^{\gamma} - 1]\}(1-\alpha)/\alpha$. Recall that $2(1/2)^{\gamma} - 1$ is equal to $\hat{A}$, which is the threshold level for linear utility that we derived in Section 2.[10]

Case 2: $A \leq [2(1/2)^{\gamma} - 1]^{\alpha}$. In this case $d(\pi/w_H)/d\delta \leq 0$, and therefore inequality (20) would become even stronger if δ were to decrease (i.e., if some workers moved from the edges to the middle). This means that the only possible equilibrium here is the one we mentioned before, in particular, the one in which all workers are located in the middle and all capital and all labor goes to the booming sector.

Case 3: $\bar{A}' > A > [2(1/2)^{\gamma} - 1]^{\alpha}$. In this case $d(\pi/w_H)/d\delta > 0$, and therefore inequality (20) becomes weaker as δ decreases. Equality in (20) will be reached for some $\delta > 0$, and thus in addition to the equilibrium in which all workers are located in the middle, there is also an equilibrium in which some workers are located at the edges and some are located at the middle.

To summarize, an economy where capital can flow across sectors after the shocks are realized, there is *less* specialization compared to an economy where such reallocation of capital is not possible. This happens because capital flows to a more productive sector, increasing wages variation even more. In addition, when the goods are perfect substitutes, there can be multiple equilibria skill distributions, varying from full specialization to full generalization (i.e., all workers acquiring both skills). Perfect substitution suggests that the good should be produced using the more efficient technology, i.e., in the good-shock sector. Even though the perfect-generalization equilibrium implies full insurance, generalizing all workers has a cost of wasting time on a skill that will not be used. Which equilibrium has a higher social welfare will depending on parameters of the model, in particular, A, θ and γ.

5 Summary and Conclusions

We study the choice of specialization under uncertainty, where the reasons for less than perfect specialization are risk-aversion, decreasing returns in human capital accumulation, and substitutability/complementarity between output products. We build a general equilibrium two-sector model, where sector-specific skills are used to produce goods, and risk-averse workers value the

[10] Remember that we consider $\psi = \infty$, and therefore $\frac{\psi-1}{\psi} = 1$.

goods produced in the two sectors according a CES utility function. Risk comes into the economy through sector-specific productivity shocks.

In this model we study a competitive equilibrium, the first best, and the constrained optimum (where no transfers among the workers can be used). For a simple case of perfectly negatively correlated productivity shocks, we show that in a competitive equilibrium for high enough variation in the shocks, a fraction of workers will generalize (acquire both skills), unless the elasticity of substitution between goods is one. In the latter case there is no wage variation, and therefore there is no reason to deviate from perfect specialization. Furthermore, when the elasticity of substitution is above (below) one, the generalists work in the good-shock (bad-shock) sector. We also establish comparative statics results, in particular, fewer workers fully specialize in competitive equilibrium if a) the risk aversion is higher, b) the education function is more concave, and c) the elasticity of substitution between two goods is further away from one.

We prove that a competitive equilibrium is generally inefficient and generates too little specialization compared to the first-best allocation. In addition, we argue that a constrained planner can improve (though not always Pareto improve) upon competitive equilibrium by reducing the degree of specialization.

In addition to the analytical results for the simple case, we provide numerical computations of equilibrium skill distributions for more general distributions of productivity shocks. While all shocks distributions result in mass points of fully specialized workers, the skill distributions are very different in other dimensions. In particular, we find that uniform perfectly negatively correlated shocks result in the density concentrated around the middle of the interval, while uniform i.i.d. shocks generate two symmetrical intervals of nonnegative density, with no workers exactly in the middle.

Finally, by analyzing modifications of the model with endogenous capital supply, we obtain that there will be more specialization (but still less than under the first best, as long as labor's share exceeds one half) if capital can flow from one sector to another prior to shocks realization, and less specialization if the capital flows ex-post.

For further research directions, it would be interesting to look at a dynamic version of this model. In particular, assuming that sector-specific shocks follow a Markov process, one could look at predictions for specialization and labor mobility assuming, e.g., on-the-job training (so that a worker improves the skill specific to the sector he works in).

Appendix: Omitted Proofs

Proof of Lemma 1. a) The expression for wages are $w_H = \alpha L_H^{\alpha-1}$ and $w_L = p\alpha A L_L^{\alpha-1} = \alpha A^{1-1/\psi} L_H^{\alpha/\psi} L_L^{\alpha-1-\alpha/\psi}$. Using the expressions for L_H and L_L, we have $dL_H/d\delta = -\left[2\left(1/2\right)^{\gamma} - 1\right] < 0$, and $dL_L/d\delta = 1 > 0$, so that

$$\begin{aligned}
dw_H/d\delta &= -\alpha\left(1-\alpha\right) L_H^{\alpha-2} dL_H/d\delta > 0, \\
dw_L/d\delta &= \alpha A^{1-1/\psi}\frac{\alpha}{\psi} L_H^{\alpha/\psi-1} L_L^{\alpha-1-\alpha/\psi} dL_H/d\delta + \alpha A^{1-\frac{1}{\psi}}\left(\alpha-1-\alpha/\psi\right) L_H^{\alpha/\psi} L_L^{\alpha-2-\alpha/\psi} < 0.
\end{aligned}$$

b) Using the above expressions, obtain

$$\begin{aligned}
&\frac{1}{2} dw_H/d\delta + \frac{1}{2}\frac{dw_L}{d\delta} - \left(\frac{1}{2}\right)^{\gamma} dw_H/d\delta \\
=&\alpha L_H^{\alpha-2}\underbrace{dL_H/d\delta}_{<0}\underbrace{\left(\frac{1}{2}\frac{\alpha}{\psi}A^{1-1/\psi}L_H^{\alpha/\psi-\alpha+1}L_L^{\alpha-1-\alpha/\psi} + \left(1-\alpha\right)\left[\left(1/2\right)^{\gamma} - 1/2\right]\right)}_{>0} \\
&+\underbrace{\alpha A^{1-\frac{1}{\psi}}\left(\alpha-1-\alpha/\psi\right)L_H^{\alpha/\psi}L_L^{\alpha-2-\alpha/\psi}}_{<0} < 0.
\end{aligned}$$

Therefore, a change in δ results in a greater change in the mean of the income for those who generalize than for those who specialize, plus in addition the specialists suffer from an additional spread (since $dw_H/d\delta > 0$ and $dw_L/d\delta < 0$). This implies that a change δ results in a greater change in the expected utility of those in the middle (generalists) relative to those at the edges (specialists) $dU_{edges}/d\delta < dU_{middle}/d\delta$.

c) Denote $y_H \equiv w_H + \pi$, $y_L \equiv w_L + \pi$, $y_M \equiv \left(1/2\right)^{\gamma} w_H + \pi$, where $\pi = \left(1-\alpha\right)\left[L_H^{\alpha} + pAL_L^{\alpha}\right] = \left(1-\alpha\right)\left[L_H^{\alpha} + A^{(\psi-1)/\psi}L_H^{\alpha/\psi}L_L^{\alpha-\alpha/\psi}\right]$, so that

$$\begin{aligned}
&d\pi/d\delta = \alpha\left(1-\alpha\right)L_H^{\alpha-1}dL_H/d\delta \\
&\quad+\alpha(1-\alpha)\frac{1}{\psi}A^{(\psi-1)/\psi}L_H^{\alpha/\psi-1}L_L^{-\alpha/\psi+\alpha}\frac{dL_H}{d\delta} + \alpha(1-\alpha)\left(1-\frac{1}{\psi}\right)A^{(\psi-1)/\psi}L_H^{\frac{\alpha}{\psi}}L_L^{-\alpha/\psi+\alpha-1}\frac{dL_L}{d\delta} \\
&\quad=\underbrace{\alpha\left(1-\alpha\right)L_H^{\alpha-2}\frac{dL_H}{d\delta}\left(L_H + \frac{1}{\psi}A^{\frac{\psi-1}{\psi}}L_H^{\frac{\alpha}{\psi}-\alpha+1}L_L^{-\frac{\alpha}{\psi}+\alpha}\right)}_{<0} + \underbrace{\alpha(1-\alpha)\left(1-\frac{1}{\psi}\right)A^{\frac{\psi-1}{\psi}}L_H^{\frac{\alpha}{\psi}}L_L^{-\frac{\alpha}{\psi}+\alpha-1}}_{>0}.
\end{aligned}$$

It is straightforward to show that the bigger the ψ, the bigger the $d\pi/d\delta$. For $\psi = 1$ we have $d\pi/d\delta < 0$. We saw above that $dw_L/d\delta < 0 < dw_H/d\delta$. Even for $\psi = \infty$ when $d\pi/d\delta$ is the largest, we have $dw_L/d\delta + d\pi/d\delta = \alpha\left(1-\alpha\right)L_H^{\alpha-1}dL_H/d\delta + \alpha\left(1-\alpha\right)AL_L^{\alpha-1}\left(L_L-1\right)/L_L < 0$. Further, even for $\psi = \infty$ (and thus also for $\psi < \infty$), $\left|dy_L/d\delta\right| \geq \left|dy_H/d\delta\right|$ because $\left(dy_H/d\delta + dy_L/d\delta\right)/\left(1-\alpha\right) = \alpha L_H^{\alpha-2}dL_H/d\delta\left(2L_H-1\right) + \alpha AL_L^{\alpha-2}\left(2L_L-1\right) \leq 0$, since $L_H \geq 1/2$ and $L_L \leq 1/2$ (by evaluating L_L and L_H at $\delta = 1/2$). In addition, by concavity of u, $u'\left(y_H\right) < u'\left(y_L\right)$. Therefore, $dU_{edges}/d\delta = \left(1/2\right)u'\left(y_H\right)dy_H/d\delta + \left(1/2\right)u'\left(y_L\right)dy_L/d\delta < 0$.

d) We know that $sign\left(dU_{middle}/d\delta\right) = sign\left(dy_M/d\delta\right)$. We have $dy_M/d\delta = (1/2)^{\gamma} dw_H/d\delta + d\pi/d\delta$, and thus the bigger the ψ, the bigger the $dy_M/d\delta$. The expression for $dy_M/d\delta$ is

$$
\begin{aligned}
dy_M/d\delta = {} & \alpha\left(1-\alpha\right) L_H^{\alpha-2} \underbrace{dL_H/d\delta}_{<0} \left(-\delta\left[2\left(1/2\right)^{\gamma}-1\right] + \frac{1}{\psi} A^{(\psi-1)/\psi} L_H^{\alpha/\psi-\alpha+1} L_L^{-\alpha/\psi+\alpha}\right) \\
& + \alpha(1-\alpha)\left(1-\frac{1}{\psi}\right) A^{(\psi-1)/\psi} L_H^{\alpha/\psi-\alpha+1} L_L^{-\alpha/\psi+\alpha} \\
& = \alpha\left(1-\alpha\right)\delta\left[2\left(1/2\right)^{\gamma}-1\right] \left\{ \underbrace{L_H^{\alpha-2}\left[2\left(1/2\right)^{\gamma}-1\right]}_{>0} + A^{(\psi-1)/\psi} L_H^{\alpha/\psi-\alpha+1} L_L^{-\alpha/\psi+\alpha} \underbrace{\left(1-2/\psi\right)}_{>0 \,\Leftrightarrow\, \psi \geq 2} \right\} \\
& + \underbrace{\alpha(1-\alpha) A^{(\psi-1)/\psi} L_H^{\alpha/\psi-\alpha+1} L_L^{-\alpha/\psi+\alpha} \left[\frac{1}{2}\right]^{\gamma} \left(1-1/\psi\right)}_{>0}.
\end{aligned}
$$

For $\psi = 1$, we have $dy_M/d\delta = \alpha\left(1-\alpha\right) L_H^{\alpha-2} dL_H/d\delta\left[2L_H - \left(1/2\right)^{\gamma}\right] < 0$, and hence $dU_{middle}/d\delta < 0$. For ψ sufficiently close to 1 (and γ sufficiently different from zero), these inequalities will continue to hold. As ψ increases enough ($\psi \geq 2$ is sufficient) we obtain $dy_M/d\delta > 0$ and thus $dU_{middle}/d\delta > 0$. ■

Proof of Lemma 2. Again, consider only the case $\psi > 1$, so that $dL_H/d\delta = -\left[2\left(1/2\right)^{\gamma}-1\right] < 0$ and $dL_L/d\delta = 1 > 0$. We saw in Lemma 1 that for ψ sufficiently close to 1 we have $dU_{edges}/d\delta < dU_{middle}/d\delta < 0$.Also, $dU_{edges}/d\delta$ remains strictly negative even for $\psi = \infty$, while $dU_{middle}/d\delta > 0$ for $\psi = \infty$. However, it will still be true that $2\delta dU_{edges}/d\delta + (1-2\delta)dU_{middle}/d\delta < 0$ at least at $\delta = \delta^{DE}$. Below is the proof of this fact for the logarithmic utility case ($\theta = 1$).

For $\theta = 1$ we can write $2\delta dU_{edges}/d\delta + (1-2\delta)dU_{middle}/d\delta = \delta[\left(1/y_H\right) dy_H/d\delta + \left(1/y_L\right) dy_L/d\delta] + \left(1-2\delta\right)\left(1/y_M\right) dy_M/d\delta$. For $\delta = \delta^{DE}$, $y_H y_L = y_M^2$, which implies that

$$
\begin{aligned}
& \alpha\left(1-\alpha\right)\left[2\delta dU_{edges}/d\delta + (1-2\delta)dU_{middle}/d\delta\right]\big|_{\delta=\delta^{DE}} \\
= & \left[\alpha\left(1-\alpha\right)/\left(y_H y_L\right)\right]\left[\delta y_L dy_H/d\delta + \delta y_H dy_L/d\delta + (1-2\delta) y_M dy_M/d\delta\right] \\
= & L_H^{\alpha-2} dL_H/d\delta\left[\delta\left(y_L + y_H\right) L_H - \delta y_L + (1-2\delta) y_M\left(L_H - \left(1/2\right)^{\gamma}\right)\right] \\
& + A L_L^{\alpha-1} \underbrace{\left[\delta\left(y_L + y_H\right) + (1-2\delta) y_M - y_H\right]}_{<0} \\
\leq & L_H^{\alpha-2} dL_H/d\delta\left[\delta\left(y_L + y_H\right) L_H + (1-2\delta) y_M L_H - \delta y_L - (1-2\delta) y_M\left(1/2\right)^{\gamma}\right],
\end{aligned}
$$

which, using the fact that at the decentralized equilibrium $\left(y_L + y_H\right)/2 \geq y_M$, is bounded by

$$
\leq L_H^{\alpha-2} dL_H/d\delta\left\{y_M\left[L_H - \left(1-2\delta\right)\left(1/2\right)^{\gamma}\right] - \delta y_L\right\} \leq L_H^{\alpha-2} \underbrace{dL_H/d\delta}_{<0}\, \delta \underbrace{\left[y_M - y_L\right]}_{>0} < 0.
$$

Therefore, $2\delta dU_{edges}/d\delta + (1-2\delta)dU_{middle}/d\delta\big|_{\delta=\delta^{DE}} < 0$. ■

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